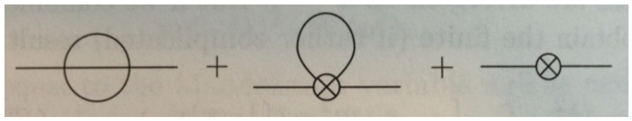
PHY2404S QFT2 HW 4 Given Θ21Mar2013 Due Θ04Apr2013

Question 1: field-strength renormalization in ϕ^4 theory

The two-loop contribution to the propagator in ϕ^4 field theory involves three diagrams as shown in Peskin and Schroeder equation (10.31):-



Using dimensional regularization, compute the first of these diagrams, the "setting sun", in the limit of zero mass for the scalar field. Show that, near $D = 4$, this graph takes the form

$$-ip^2 \frac{\lambda^2}{12(4\pi)^4}\left[-\frac{1}{\epsilon} + \log(p^2) + \cdots\right] \tag{1}$$

with $\epsilon = 4 - D$. The coefficient in this equation involves a Feynman parameter integral that can be evaluated by setting $D = 4$. Verify that the second diagram in (10.31) vanishes near $D = 4$. Thus the first diagram should contain a pole only at $\epsilon = 0$, which can be cancelled by a field-strength renormalization counterterm.

Question 2: conceptual exploration

Write 1.5-2 pages answering the following questions in your own words. Include examples.

(a) What is a critical exponent?

(b) What is the Coleman-Mermin-Wagner theorem?

Question 3: beta functions in Yukawa theory with a pseudoscalar

In the pseudoscalar Yukawa theory described by

$$\mathscr{L} = \frac{1}{2}(\partial\phi)^2 + \bar{\psi}(i\partial\!\!\!/)\psi - \frac{\lambda}{4!}\phi^4 - ig\bar{\psi}\gamma_5\psi\phi\,, \tag{2}$$

compute the Callan-Symanzik β-functions for λ and g, namely

$$\beta_\lambda(\lambda, g) \quad \text{and} \quad \beta_g(\lambda, g)\,, \tag{3}$$

to leading order in coupling constants, assuming that λ and g^2 are of the same order. Sketch the coupling constant flows in the $\lambda{-}g$ plane.

(Note: this is a pretty long problem, and it is good preparation for the final exam, so please set aside plenty of time to complete it.)
